SADLER MATHEMATICS METHODS UNIT 1

WORKED SOLUTIONS

Chapter 6 Quadratic equations

Exercise 6A

Question 1

$(x+5)(x-3)=0$

Either $x+5=0$ or $x-3=0$

$x=-5$ $x=3$

Question 2

$(x+8)(x+9)=0$

Either $x+8=0$ or $x+9=0$

$x=-8$ $x=-9$

Question 3

$(2x-11)(x+5)=0$

Either $2x-11=0$ or $x+5=0$

$x=5.5$ $x=-5$

Question 4

$x^2=25$

$x=\pm 5$

Question 5

$$x^2 - 49 = 0$$
$$(x+7)(x-7) = 0$$
$$x = \pm 7$$

Question 6

$$2x^2 = 200$$
$$x^2 = 100$$
$$x = \pm 10$$

Question 7

$$x^2 + 9x + 20 = 0$$
$$(x+5)(x+4) = 0$$
Either $x+5=0$ or $x+4=0$
$$x = -5 \qquad x = -4$$

Question 8

$$x^2 + x - 20 = 0$$
$$(x+5)(x-4) = 0$$
Either $x+5=0$ or $x-4=0$
$$x = -5 \qquad x = 4$$

Question 9

$$x^2 - 9x + 20 = 0$$
$$(x-5)(x-4) = 0$$
Either $x-5=0$ or $x-4=0$
$$x = 5 \qquad x = 4$$

Question 10

$$x^2 - x - 20 = 0$$
$$(x-5)(x+4) = 0$$
Either $x-5=0$ or $x+4=0$
$x=5$ $x=-4$

Question 11

$$x^2 + 2x - 35 = 0$$
$$(x-5)(x+7) = 0$$
Either $x-5=0$ or $x+7=0$
$x=5$ $x=-7$

Question 12

$$x^2 + 4x + 3 = 0$$
$$(x+3)(x+1) = 0$$
Either $x+3=0$ or $x+1=0$
$x=-3$ $x=-1$

Question 13

$$x^2 + 7x + 6 = 0$$
$$(x+6)(x+1) = 0$$
Either $x+6=0$ or $x+1=0$
$x=-6$ $x=-1$

Question 14

$$x^2 + 10x + 21 = 0$$
$$(x+3)(x+7) = 0$$
Either $x+3=0$ or $x+7=0$
$x=-3$ $x=-7$

Question 15

$$x^2+8x+15=0$$
$$(x+3)(x+5)=0$$
Either $x+3=0$ or $x+5=0$
$x=-3$ $x=-5$

Question 16

$$x^2-4x-12=0$$
$$(x-6)(x+2)=0$$
Either $x-6=0$ or $x+2=0$
$x=6$ $x=-2$

Question 17

$$x^2-4x-5=0$$
$$(x-5)(x+1)=0$$
Either $x-5=0$ or $x+1=0$
$x=5$ $x=-1$

Question 18

$$x^2-4x=0$$
$$x(x-4)=0$$
Either $x=0$ or $x-4=0$
$x=4$

Question 19

$$x^2+5x-14=0$$
$$(x-2)(x+7)=0$$
Either $x-2=0$ or $x+7=0$
$x=2$ $x=-7$

Question 20

$$x^2 - 36 = 0$$
$$(x+6)(x-6) = 0$$
$$x = \pm 6$$

Question 21

$$x^2 + 6x + 9 = 0$$
$$(x+3)^2 = 0$$
$$x + 3 = 0$$
$$x = -3$$

Question 22

$$x^2 - 3x - 4 = 0$$
$$(x-4)(x+1) = 0$$

Either $x - 4 = 0$ or $x + 1 = 0$

$$x = 4 \qquad x = -1$$

Question 23

$$x^2 - 8x + 16 = 0$$
$$(x-4)^2 = 0$$
$$x - 4 = 0$$
$$x = 4$$

Question 24

$$x^2 = 15 - 2x$$
$$x^2 + 2x - 15 = 0$$
$$(x-3)(x+5) = 0$$
Either $x - 3 = 0$ or $x + 5 = 0$
$$x = 3 \qquad x = -5$$

Question 25

$$x^2 = 3x$$
$$x^2 - 3x = 0$$
$$x(x-3) = 0$$
Either $x = 0$ or $x - 3 = 0$
$$x = 3$$

Question 26

$$x^2 + 12 = 7x$$
$$x^2 - 7x + 12 = 0$$
$$(x-3)(x-4) = 0$$
Either $x - 3 = 0$ or $x - 4 = 0$
$$x = 3 \qquad x = 4$$

Question 27

$$x^2 = 24 - 10x$$
$$x^2 + 10x - 24 = 0$$
$$(x-2)(x+12) = 0$$
Either $x - 2 = 0$ or $x + 12 = 0$
$$x = 2 \qquad x = -12$$

Question 28

$$4x^2 - 9 = 0$$
$$(2x+3)(2x-3) = 0$$
Either $2x+3=0$ or $2x-3=0$
$$x = \pm 1.5$$

Question 29

$$25x^2 - 1 = 0$$
$$(5x-1)(5x+1) = 0$$
Either $5x-1=0$ or $5x+1=0$
$$x = \pm 0.2$$

Question 30

$$x^2 = 2x + 15$$
$$x^2 - 2x - 15 = 0$$
$$(x-5)(x+3) = 0$$
Either $x-5=0$ or $x+3=0$
$$x = 5 \qquad x = -3$$

Question 31

$$x^2 + 9 = 6x$$
$$x^2 - 6x + 9 = 0$$
$$(x-3)^2 = 0$$
$$x - 3 = 0$$
$$x = 3$$

Question 32

$$x^2 = 5(2x-5)$$
$$x^2 - 10x + 25 = 0$$
$$(x-5)^2 = 0$$
$$x - 5 = 0$$
$$x = 5$$

Question 33

$$2x^2 + 5x - 12 = 0$$
$$(2x-3)(x+4) = 0$$
Either $2x-3=0$ or $x+4=0$
$$x = 1.5 \qquad x = -4$$

Question 34

$$3x^2 + 10x - 8 = 0$$
$$(3x-2)(x+4) = 0$$
Either $3x-2=0$ or $x+4=0$
$$x = \frac{2}{3} \qquad x = -4$$

Question 35

$$2x^2 - 3x - 5 = 0$$
$$(2x-5)(x+1) = 0$$
Either $2x-5=0$ or $x+1=0$
$$x = 2.5 \qquad x = -1$$

Question 36

$5x^2 - 34x - 7 = 0$

$(5x + 1)(x - 7) = 0$

Either $5x + 1 = 0$ or $x - 7 = 0$

$x = -0.2$ $x = 7$

Question 37

$2x^2 + x - 21 = 0$

$(2x + 7)(x - 3) = 0$

Either $2x + 7 = 0$ or $x - 3 = 0$

$x = -3.5$ $x = 3$

Question 38

$6x^2 - 19x + 10 = 0$

$(3x - 2)(2x - 5) = 0$

Either $3x - 2 = 0$ or $2x - 5 = 0$

$x = \frac{2}{3}$ $x = 2.5$

Question 39

$10x^2 - 9x + 2 = 0$

$(2x - 1)(5x - 2) = 0$

Either $2x - 1 = 0$ or $5x - 2 = 0$

$x = 0.5$ $x = 0.4$

Question 40

$$x^2 + 7x = 30$$
$$x^2 + 7x - 30 = 0$$
$$(x+10)(x-3) = 0$$
Either $x+10=0$ or $x-3=0$
$$x = -10 \qquad x = 3$$

The number is either –10 or 3.

Question 41

$$x^2 + 10x + 25 = 0$$
$$(x+5)^2 = 0$$
$$x+5=0$$
$$x=-5$$

The number is –5.

Question 42

When the object hits the ground, $h = 0$.
$$h = 40t - 5t^2 = 0$$
$$5t(8-t) = 0$$
Either $5t=0$ or $8-t=0$
$$t=0 \qquad t=8$$

Question 43

$$s = ut + \frac{1}{2}at^2$$

$$10 = 3t + \frac{1}{2}(2)t^2$$

$$t^2 + 3t - 10 = 0$$

$$(t-2)(t+5) = 0$$

Either $t - 2 = 0$ or $t + 5 = 0$

$t = 2$ $\quad$ $t = -5$

Given $t \geq 0, t = 2$

Question 44

$$w = kp^2 - 2cp$$

$$33 = 1p^2 - 2(4)p$$

$$p^2 - 8p - 33 = 0$$

$$(p-11)(p+3) = 0$$

Either $p - 11 = 0$ or $p + 3 = 0$

$p = 11$ $\quad$ $p = -3$

Exercise 6B

Question 1

$x = -0.77,\ 0.43$

Question 2

$x = -2.30,\ 1.30$

Question 3

No real solutions

Question 4

$x = -2.82, -0.18$

Question 5

$x = -1.74, 0.34$

Question 6

$x = -1.47,\ 0.27$

Question 7

$t = 13.8$

Question 8

$p = 0.22,\ 2.78$

Question 9

Graph does not touch x-axis therefore no real solutions exist.

Question 10

Graph cuts x-axis twice therefore two real solutions exist.

Question 11

Graph does not touch x-axis therefore no real solutions exist.

Question 12

Graph is tangent to x-axis therefore one real solution exists.

Question 13

Graph is tangent to x-axis therefore one real solution exists.

Question 14

Graph cuts x-axis twice therefore two real solutions exist.

Question 15

Curve and line intersect twice, so two real solutions exist.

Question 16

Curve and line do not intersect at all therefore no real solutions exist.

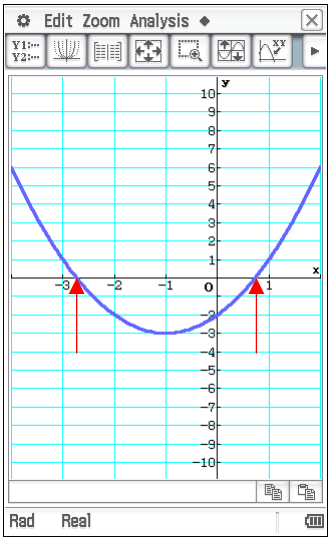
Question 17

Line is tangent to the curve therefore one real solution exists.

Question 18

By inspection, $x \approx -2.7, 0.7$

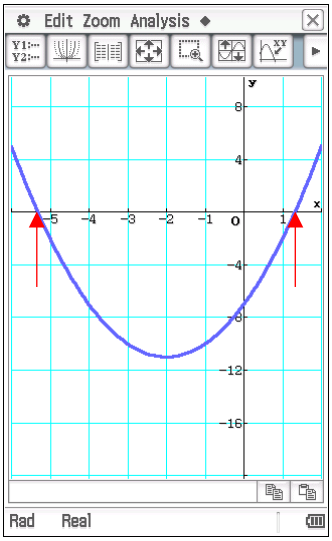
(Accuracy will depend on sketch)



Question 19

By inspection, $x \approx -5.3, 1.3$

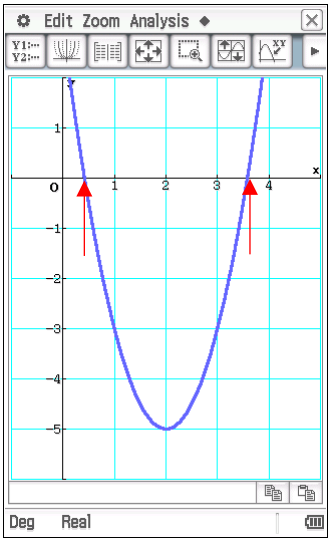
(Accuracy will depend on sketch)



Question 20

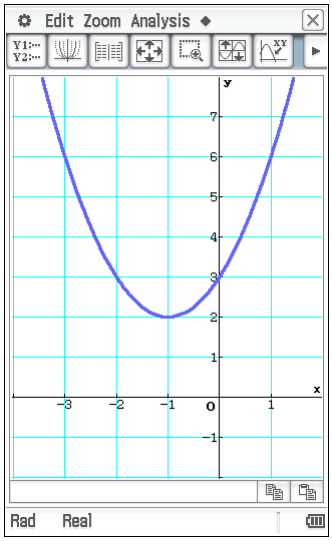
By inspection, $x \approx 0.4, 3.6$

(Accuracy will depend on sketch)



Question 21

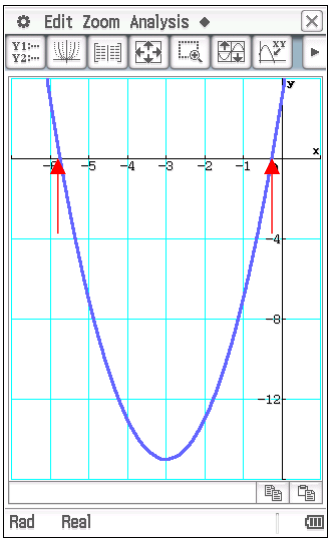
By inspection, there are no solutions as the graph does not cross the x-axis.



Question 22

By inspection, $x \approx -5.7, -0.3$

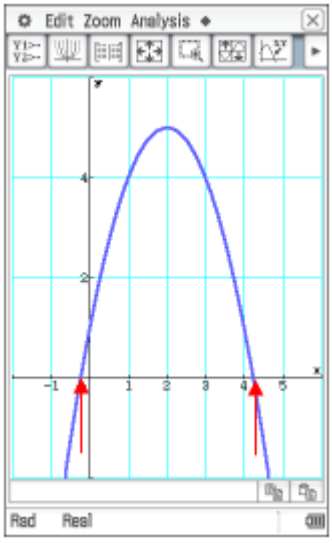
(Accuracy will depend on sketch)



Question 23

By inspection, $x \approx -0.2, 4.2$

(Accuracy will depend on sketch)



Question 24

$$(x-6)^2-36+21=0$$
$$(x-6)^2-15=0$$
$$(x-6)^2=15$$
$$x-6=\pm\sqrt{15}$$
$$x=\pm\sqrt{15}+6$$
$$=2.13,\ 9.87$$

Question 25

$$(x-3)^2-9+10=0$$
$$(x-6)^2+1=0$$
$$(x-6)^2=-1$$

No real solutions exist

Question 26

$$(x-4)^2-16+1=0$$
$$(x-4)^2-15=0$$
$$(x-4)^2=15$$
$$x-4=\pm\sqrt{15}$$
$$x=\pm\sqrt{15}+4$$
$$=0.13,\ 7.87$$

Question 27

$$(x+3.5)^2-12.25-5=0$$
$$(x+3.5)^2-17.25=0$$
$$(x+3.5)^2=17.25$$
$$x+3.5=\pm\sqrt{17.25}$$
$$x=\pm\sqrt{17.25}-3.5$$
$$=-7.65,\ 0.65$$

Question 28

$$(x+1.5)^2-2.25-5=0$$
$$(x+1.5)^2-7.25=0$$
$$(x+1.5)^2=7.25$$
$$x+1.5=\pm\sqrt{7.25}$$
$$x=\pm\sqrt{7.25}-1.5$$
$$=-4.19,\ 1.19$$

Question 29

$$2(x^2+\frac{1}{2}x-1.5)=0$$
$$2[(x+\frac{1}{4})^2-\frac{1}{16}-\frac{3}{2}]=0$$
$$2[(x+\frac{1}{4})^2-\frac{25}{16}]=0$$
$$(x+\frac{1}{4})^2=\frac{25}{16}$$
$$x+\frac{1}{4}=\pm\frac{5}{4}$$
$$x=-\frac{1}{4}\pm\frac{5}{4}$$
$$=-1.5,1$$

Question 30

$$(x-1)^2-1-5=0$$
$$(x-1)^2-6=0$$
$$(x-1)^2=6$$
$$x-1=\pm\sqrt{6}$$
$$x=1\pm\sqrt{6}$$

Question 31

$$(x-3)^2-9+1=0$$
$$(x-2)^2-8=0$$
$$(x-3)^2=8$$
$$x-3=\pm\sqrt{8}$$
$$x=3\pm2\sqrt{2}$$

Question 32

$$(x+5)^2-25-7=0$$
$$(x+5)^2-32=0$$
$$(x+5)^2=32$$
$$x+5=\pm\sqrt{32}$$
$$x=-5\pm4\sqrt{2}$$

Question 33

$$2\left(x^2+5x-2.5\right)=0$$
$$2\left[\left(x+2.5\right)^2-6.25-2.5\right]=0$$
$$2\left[\left(x+2.5\right)^2-8.75\right]=0$$
$$\left(x+2.5\right)^2-\tfrac{35}{4}=0$$
$$x+2.5=\pm\sqrt{\tfrac{35}{4}}$$
$$x=-\frac{5}{2}\pm\frac{\sqrt{35}}{2}$$

Question 34

$$3\left(x^2+\tfrac{5}{3}x+\tfrac{1}{3}\right)=0$$
$$3\left[\left(x+\tfrac{5}{6}\right)^2-\tfrac{25}{36}+\tfrac{1}{3}\right]=0$$
$$3\left[\left(x+\tfrac{5}{6}\right)^2-\tfrac{13}{36}\right]=0$$
$$\left(x+\tfrac{5}{6}\right)^2-\tfrac{13}{36}=0$$
$$x+\tfrac{5}{6}=\pm\sqrt{\tfrac{13}{36}}$$
$$x=-\frac{5}{6}\pm\frac{\sqrt{13}}{6}$$

Question 35

$$5\left(x^2+\tfrac{1}{5}x-\tfrac{1}{5}\right)=0$$
$$3\left[\left(x+\tfrac{1}{10}\right)^2-\tfrac{1}{100}-\tfrac{1}{5}\right]=0$$
$$3\left[\left(x+\tfrac{1}{10}\right)^2-\tfrac{21}{100}\right]=0$$
$$\left(x+\tfrac{1}{10}\right)^2-\tfrac{21}{100}=0$$
$$x+\tfrac{1}{10}=\pm\sqrt{\tfrac{21}{100}}$$
$$x=-\frac{1}{10}\pm\frac{\sqrt{21}}{10}$$

Question 36

$$a=1,\ b=1,\ c=-4$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-1\pm\sqrt{1^2-4(1)(-4)}}{2}$$
$$=\frac{-1\pm\sqrt{17}}{2}$$
$$=-2.56, 1.56$$

Question 37

$$a=-2,\ \ b=7,\ c=5$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-7\pm\sqrt{7^2-4(-2)(5)}}{2(-2)}$$
$$=\frac{-7\pm\sqrt{89}}{-4}$$
$$=-0.61, 4.11$$

Question 38

$3x^2 + 1 = 7x$

$3x^2 - 7x + 1 = 0$

$a = 3, \ b = -7, \ c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{7 \pm \sqrt{(-7)^2 - 4(3)(1)}}{2(3)}$$

$$= \frac{7 \pm \sqrt{37}}{6}$$

$$= 0.15, \ 2.18$$

Question 39

$6x = x^2 + 7$

$x^2 - 6x + 7 = 0$

$a = 1, \ b = -6, \ c = 7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{8}}{2}$$

$$= 1.59, \ 4.41$$

Question 40

$$x(x-1)=7$$
$$x^2-x-7=0$$

$$a=1,\ \ b=-1,\ \ c=-7$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{1\pm\sqrt{(-1)^2-4(1)(-7)}}{2(1)}$$
$$=\frac{1\pm\sqrt{29}}{2}$$
$$=-2.19,\ 3.19$$

Question 41

$$2x(3x+1)=5$$
$$6x^2+2x-5=0$$

$$a=6,\ \ b=2,\ \ c=-5$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-2\pm\sqrt{(-2)^2-4(6)(-5)}}{2(6)}$$
$$=\frac{-2\pm\sqrt{124}}{12}$$
$$=-1.09,0.76$$

Question 42

$a = 1,\ \ b = 3,\ \ c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-3 \pm \sqrt{(3)^2 - 4(1)(1)}}{2(1)}$$
$$= \frac{-3 \pm \sqrt{5}}{2}$$
$$= -\frac{3}{2} \pm \frac{\sqrt{5}}{2}$$

Question 43

$a = 1,\ \ b = -7,\ \ c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{7 \pm \sqrt{(-7)^2 - 4(1)(1)}}{2(1)}$$
$$= \frac{7 \pm \sqrt{45}}{2}$$
$$= \frac{7}{2} \pm \frac{3\sqrt{5}}{2}$$

Question 44

$a = 2,\ \ b = 1,\ \ c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-1 \pm \sqrt{(1)^2 - 4(2)(-5)}}{2(2)}$$
$$= \frac{-1 \pm \sqrt{41}}{4}$$
$$= -\frac{1}{4} \pm \frac{\sqrt{41}}{4}$$

Question 45

$3x^2 = 1 + 5x$

$3x^2 - 5x - 1 = 0$

$a = 3, \ \ b = -5, \ \ c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{5 \pm \sqrt{(-5)^2 - 4(3)(-1)}}{2(6)}$$

$$= \frac{5 \pm \sqrt{37}}{6}$$

$$= \frac{5}{6} \pm \frac{\sqrt{37}}{6}$$

Question 46

$a = 5, \ \ b = 1, \ \ c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{(1)^2 - 4(5)(-5)}}{2(5)}$$

$$= \frac{-1 \pm \sqrt{101}}{10}$$

$$= -\frac{1}{10} \pm \frac{\sqrt{101}}{10}$$

Question 47

$2x(x+2) = -1$

$2x^2 + 4x + 1 = 0$

$a = 2, \ \ b = 4, \ c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{(4)^2 - 4(2)(1)}}{2(2)}$$

$$= \frac{-4 \pm \sqrt{8}}{4}$$

$$= -1 \pm \frac{\sqrt{2}}{2}$$

Question 48

$a = 1, \ b = 5, \ c = -7$

$\Delta = b^2 - 4ac$

$= 5^2 - 4(1)(-7)$

$= 53$

$\Delta > 0$ therefore equation has two real roots

Question 49

$a = 1, \ b = 5, \ c = 7$

$\Delta = b^2 - 4ac$

$= 5^2 - 4(1)(7)$

$= -3$

$\Delta < 0$ therefore equation has no real roots

Question 50

$a = 1,\ b = -2,\ c = -3$

$\Delta = b^2 - 4ac$

$= (-2)^2 - 4(1)(-3)$

$= 16$

$\Delta > 0$ therefore equation has two real roots

Question 51

$a = 2,\ b = 7,\ c = 5$

$\Delta = b^2 - 4ac$

$= (7)^2 - 4(2)(5)$

$= 9$

$\Delta > 0$ therefore equation has two real roots

Question 52

$a = 4,\ b = -12,\ c = 9$

$\Delta = b^2 - 4ac$

$= (-12)^2 - 4(4)(9)$

$= 0$

$\Delta = 0$ therefore equation has one real root

Question 53

$a = 3,\ b = -1,\ c = 1$

$\Delta = b^2 - 4ac$

$= (-1)^2 - 4(3)(1)$

$= -11$

$\Delta < 0$ therefore equation has no real roots

Question 1

$(x-3)(x+5)=0$

$x-3=0$ or $x+5=0$

$x=3$ $\qquad$ $x=-5$

Question 2

Line A has a y-intercept of (0, 60) and a negative gradient $\Rightarrow y=-x+60$

Line B is a horizontal line $\Rightarrow y=60$

Line C has a negative y-intercept and a positive gradient $\Rightarrow y=2x-60$

Line D is a vertical line $\Rightarrow x=60$

Line E has a y-intercept of (0, 30) and a negative gradient $\Rightarrow y=-2x+30$

Line F has a y-intercept of (0, 30) and a positive gradient $\Rightarrow y=0.5x+30$

Question 3

a Point D has co-ordinates (6, 5), Point E (14,5) and Point F (14, 11).

AD = 6 units, DB = 6 units.

$$m_{AB}=\frac{6}{6}=1$$

b DE = 8 units, EC = 4 units.

$$m_{DC}=\frac{4}{8}=\frac{1}{2}$$

c DE = 8 units, EF= 6 units.

$$m_{DF}=\frac{6}{8}=\frac{3}{4}$$

Question 4

$y = 3(x-1)^2 + 2$

line of symmetry $x = 1 \therefore a = 1$

turning poin t $(1,2) \therefore b = 1, c = 2$

$d = 3(6-1)^2 + 2 = 77$

$e = 3(-4-1)^2 + 2 = 77$

$14 = 3(f-1)^2 + 2$

$3(f-1)^2 = 12$

$(f-1)^2 = 4$

$f - 1 = \pm 2$

$f = -1, 3$

Question 5

$5w = l - 3 \qquad \Rightarrow l = 5w + 3$

$A = l \times w$

$36 = (5w+3)w$

$5w^2 + 3w - 36 = 0$

$(5w-12)(w+3) = 0$

$5w = 12$ or $w + 3 = 0$

$w = 2.4 \qquad w = -3$

Dis regard $w = -3$ as width cannot be negative

$\therefore w = 2.4\,\text{cm}$

$l = 5(2.4) + 3 = 15\,\text{cm}$

Question 6

Area of triangle

$\frac{1}{2} \times 5 \times 10 = 25 \text{ cm}^2$

$\tan \angle AOB = \frac{10}{5} = 2$

$\angle AOB = 1.107 \text{ radians}$

Area of sector AOB

$\frac{1}{2} 5^2 (1.107)$

$= 13.84 \text{ cm}^2$

Area of triangle outside circle

$25 - 13.84 \text{ cm}^2$

$= 11.2 \text{ cm}^2$ (to 1 dp)

Question 7

Curve A

minimum tp (0,-1) $\therefore y = ax^2 - 1$

x-int (1,0)

$0 = a(1)^2 - 1$

$0 = a - 1$

$a = 1$

Equation of A : $y = x^2 - 1$

Curve B

minimum tp (7,0) $\therefore y = a(x-7)^2$

B passes through (8,1)

$1 = a(8-7)^2$

$1 = a$

Equation of B : $y = (x-7)^2$

Curve C

minimum tp (-9,2) $\therefore y = a(x+9)^2 + 2$

C passes through (-8,3)

$3 = a(-8+9)^2 + 2$

$1 = a$

Equation of C : $y = (x+9)^2 + 2$

Curve D

minimum tp (- 5, -8) $\therefore y = a(x+5)^2 - 8$

D passes through $(-4,-7)$

$-7 = a(-4+5)^2 - 8$

$1 = a$

Equation of D : $y = (x+5)^2 - 8$

Curve E

maximum tp (4,1) $\therefore y = -a(x-4)^2 + 1$

E passes through $(5,0)$

$0 = -a(5-4)^2 + 1$

$-1 = -a$

$a = 1$

Equation of E : $y = -(x-4)^2 + 1$

Curve F

minimum tp (10,0) $\therefore y = a(x-10)^2$

F passes through $(11,2)$

$2 = a(11-10)^2$

$a = 2$

Equation of F : $y = 2(x-10)^2$

Curve G

minimum tp (-5,-3) $\therefore y = a(x+5)^2 - 3$

G passes through $(-4,1)$

$1 = a(-4+5)^2 - 3$

$4 = a$

Equation of G : $y = 4(x+5)^2 - 3$

Curve H

maximum tp $(-10,0)$ $\therefore y = -a(x+10)^2$

H passes through $(-9,-2)$

$-2 = -a(-9+10)^2$

$-2 = -a$

$a = 2$

Equation of H : $y = -2(x+10)^2$

Question 8

$SA = 2\pi r^2 + 2\pi rh, \quad SA = 2000, h = 30$

$2000 = 2\pi r^2 + 2\pi r(30)$

$2\pi r^2 + 60\pi r - 2000 = 0$

$a = 2\pi,\ b = 60\pi,\ c = -2000$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-60\pi \pm \sqrt{(60\pi)^2 - 4(2\pi)(-2000)}}{2(2\pi)}$$

$= -38.31, 8.31$

Disregard -38.31 as the radius cannot be negative

$\therefore r = 8.3$ cm

Question 9

$$(x+7)^2+(x+3)^2=(x+12)^2$$
$$x^2+14x+49+x^2+6x+9=x^2+24x+144$$
$$x^2-4x-86=0$$
$$(x-2)^2-4-86=0$$
$$(x-2)^2=90$$
$$x-2=\pm\sqrt{90}$$
$$x=2\pm3\sqrt{10}$$
$$=-7.49, 11.49$$

A negative solution is not a sensible solution in this situation

$\therefore x=11.49$

Question 10

$$ax^2+bx+c=0$$
$$a(x^2+\tfrac{b}{a}x+\tfrac{c}{a})=0$$
$$a\left[\left(x+\tfrac{b}{2a}\right)^2-\tfrac{b^2}{4a^2}+\tfrac{c}{a}\right]=0$$
$$\left(x+\tfrac{b}{2a}\right)^2-\tfrac{b^2}{4a^2}+\tfrac{c}{a}=0$$
$$\left(x+\tfrac{b}{2a}\right)^2=\tfrac{b^2}{4a^2}-\tfrac{c}{a}$$
$$\left(x+\tfrac{b}{2a}\right)^2=\tfrac{b^2-4ac}{4a^2}$$
$$x+\tfrac{b}{2a}=\pm\sqrt{\tfrac{b^2-4ac}{4a^2}}$$
$$=\pm\tfrac{\sqrt{b^2-4ac}}{2a}$$
$$x=-\tfrac{b}{2a}\pm\tfrac{\sqrt{b^2-4ac}}{2a}$$
$$=\tfrac{-b\pm\sqrt{b^2-4ac}}{2a}$$